

# Measuring Skewness Premia

## Online Appendix

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# 1. Cross-sectional distributions of firm risk measures and characteristics

In this section, we report the time-series of quantiles of variables used in the predictive panel regression. Fig. 1 reports on coskewness while Fig. 2 reports on all predictors.

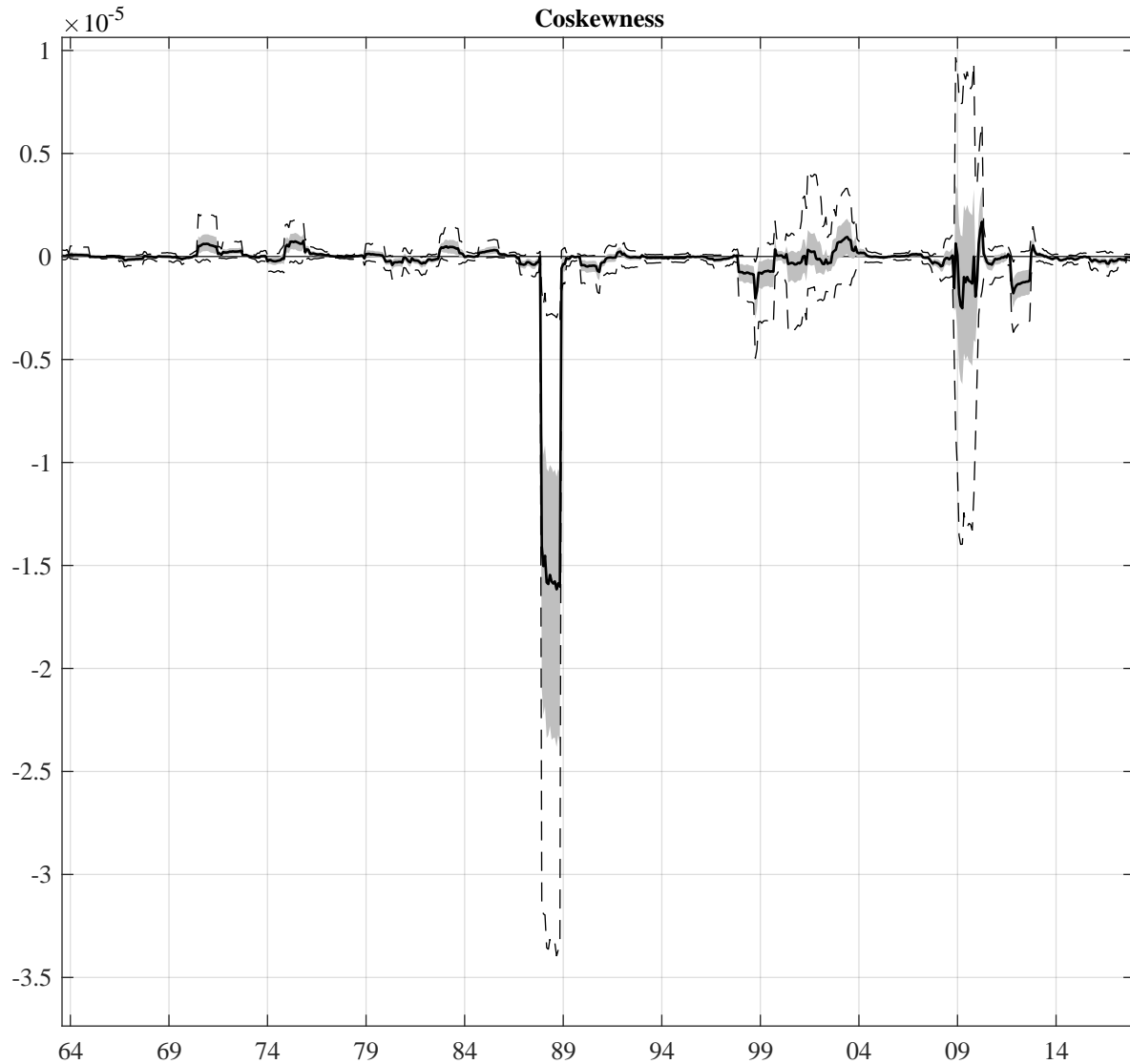
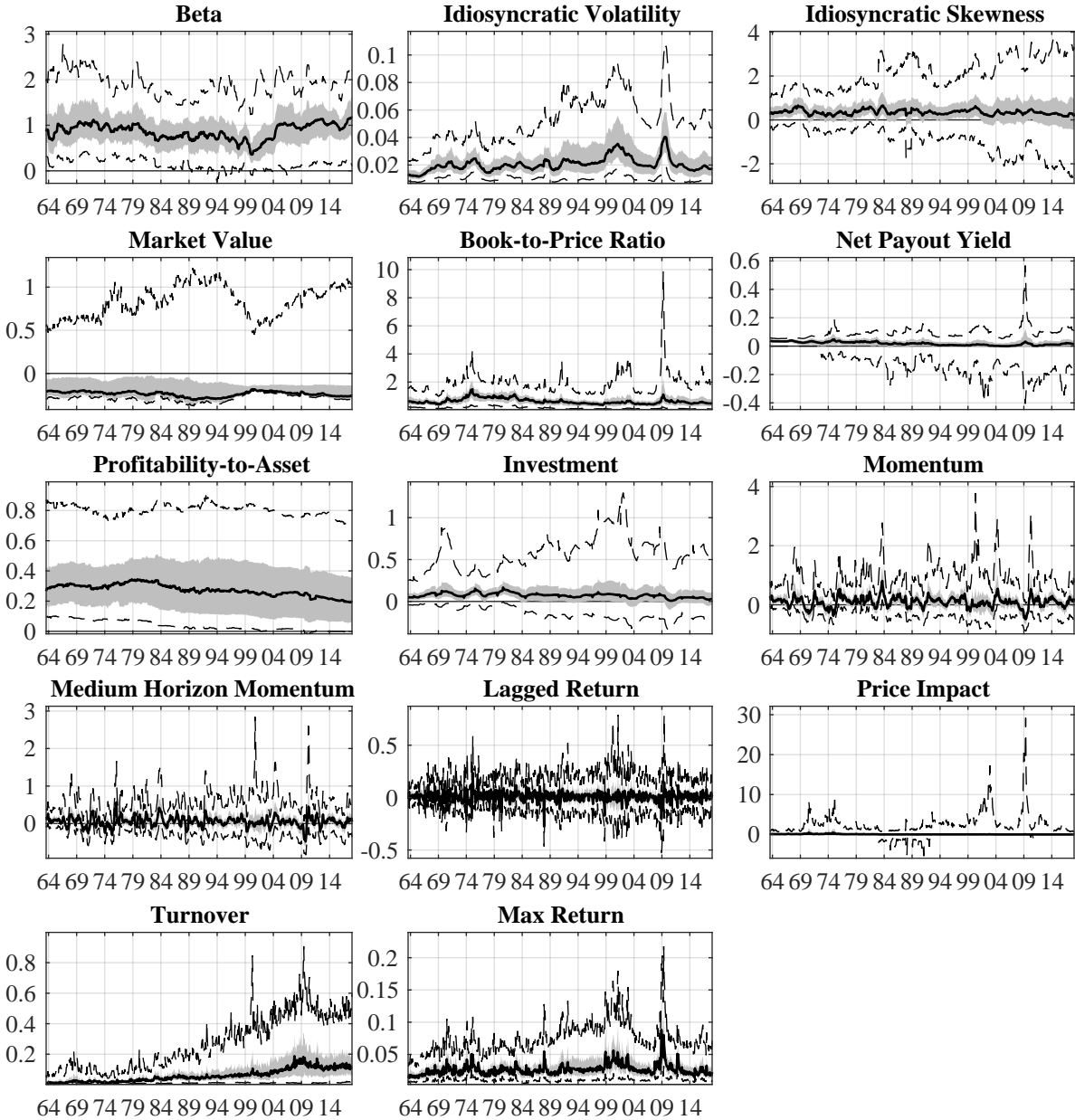


Fig. 1. **Cross-sectional quantiles of daily coskewness**

We report each month  $t$  the cross-sectional quantiles of daily coskewness  $Cos_{i,t}$ . We measure daily coskewness using daily returns from month  $t - 12$  to  $t - 1$ . We report the median using a black line, the 25<sup>th</sup>-75<sup>th</sup> interquartile range using a shaded gray area, and the 5<sup>th</sup> and 95<sup>th</sup> percentiles using dashed lines. The sample period is July 1963 to December 2017.



**Fig. 2. Cross-sectional quantiles of risk measures and characteristics**

We report each month  $t$  the cross-sectional quantiles of firm risk measures and characteristics. We measure daily risk measures using daily returns from month  $t - 12$  to  $t - 1$ . We report the median using a black line, the 25<sup>th</sup>-75<sup>th</sup> interquartile range using a shaded gray area, and the 5<sup>th</sup> and 95<sup>th</sup> percentiles using dashed lines. We standardize each month, the market capitalizations using their cross-sectional average and standard deviation. The construction of all variables is detailed in Appendix A of the main text. The sample period is July 1963 to December 2017.

## 2. Summary statistics for panel regression coefficients to predict coskewness ranks

In this section, we report additional summary statistics on estimated coefficients in the predictive panel regression (see Eq. (4) in the main text) used to forecast coskewness ranks.

Table 1: Summary statistics for panel regression coefficients to predict the coskewness ranks

	Average	5 <sup>th</sup> percentile	95 <sup>th</sup> percentile
Market beta $\beta_M$	0.036	-0.022	0.086
Idiosyncratic volatility	-0.059	-0.085	-0.028
Coskewness $Cos$	0.059	0.041	0.074
Idiosyncratic skewness	-0.019	-0.031	0.007
Market capitalization	0.079	0.039	0.147
Book-to-price ratio	-0.000	-0.029	0.015
Net payout yield	0.009	-0.010	0.036
Profitability	-0.005	-0.026	0.010
Investment	-0.005	-0.015	0.004
Momentum	-0.081	-0.138	-0.049
Intermediate horizon return	0.028	0.016	0.062
Lagged monthly return	-0.052	-0.066	-0.038
Price impact	-0.039	-0.073	0.009
Turnover	-0.036	-0.069	0.018
Maximum return	0.049	0.023	0.064

We report summary statistics of regression coefficients  $\hat{\theta}$  and  $\hat{\phi}$  in Eq. (4) in the main text from July 1963 to December 2017. We compute the time-series average and 5<sup>th</sup> and 95<sup>th</sup> percentiles. Each month, we run a panel regression that predicts the cross-sectional rank of the daily coskewness computed over the next year using past risk measures and stock characteristics. We use the cross-sectional rank of coskewness as the dependent variable and the cross-sectional ranks of past risk measures and characteristics as predictors. We use all past observations to estimate each point in time in the panel regression. The construction of all variables is detailed in Appendix A of the main text.

### 3. Summary statistics for panel regression coefficients to predict idiosyncratic skewness ranks

In this section, we report additional summary statistics on estimated coefficients in the predictive panel regression (see Eq. (4) in the main text) used to forecast idiosyncratic skewness ranks.

Table 2: Summary statistics for panel regression coefficients to predict the idiosyncratic skewness ranks

	Average	5 <sup>th</sup> percentile	95 <sup>th</sup> percentile
Market beta $\beta_M$	0.040	0.020	0.059
Idiosyncratic volatility	0.066	-0.009	0.109
Coskewness $Cos$	-0.012	-0.026	0.017
Idiosyncratic skewness	0.135	0.122	0.144
Market capitalization	-0.249	-0.340	-0.221
Book-to-price ratio	0.043	0.026	0.074
Net payout yield	-0.036	-0.121	-0.007
Profitability	-0.014	-0.025	0.024
Investment	-0.035	-0.046	-0.025
Momentum	-0.073	-0.100	-0.026
Intermediate horizon return	-0.005	-0.029	0.007
Lagged monthly return	-0.021	-0.033	-0.010
Price impact	-0.029	-0.091	-0.005
Turnover	-0.008	-0.044	0.012
Maximum return	-0.012	-0.036	0.037

We report summary statistics of regression coefficients  $\hat{\theta}$  and  $\hat{\phi}$  in Eq. (4) from July 1963 to December 2017. We compute the time-series average and 5<sup>th</sup> and 95<sup>th</sup> percentiles. Each month, we run a panel regression that predicts the cross-sectional rank of the daily idiosyncratic skewness computed over the next year using past risk measures and stock characteristics. We use the cross-sectional rank of idiosyncratic skewness as the dependent variable and the cross-sectional ranks of past risk measures and characteristics as predictors. We use all past observations to estimate at each point in time the panel regression. The construction of all variables is detailed in Appendix A of the main text.

## 4. Predicted total skewness ranks

In this section, we present results on predicted total skewness ranks. Fig. 3 reports the time-series of estimated coefficients in the panel regression used to forecast total skewness ranks. Table 3 presents the coefficients' time-series averages and 5<sup>th</sup> and 95<sup>th</sup> percentiles. Fig. 4 presents the average stock-specific realized total skewness based on the predictive panel regression and based on other skewness predictors.

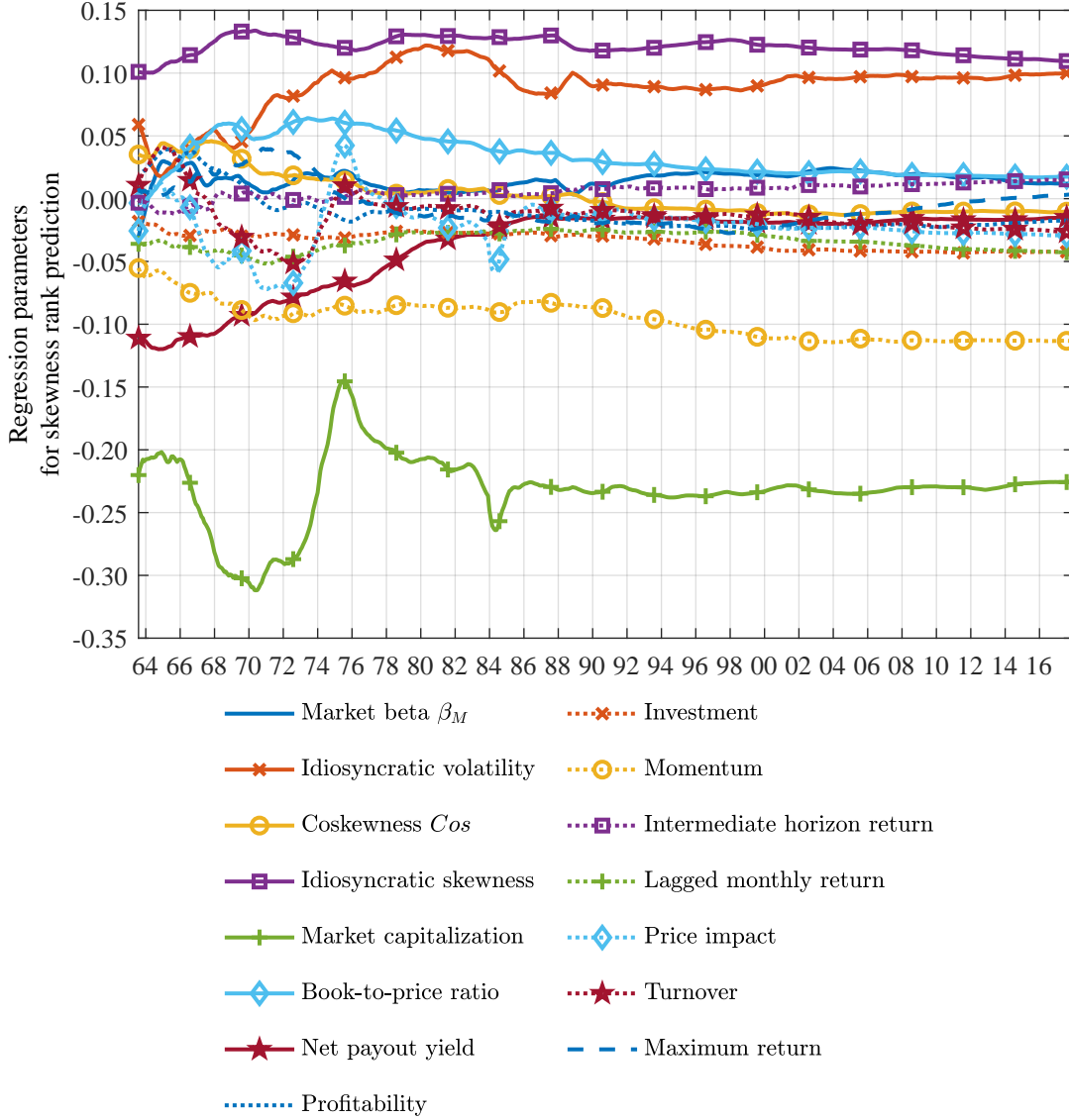


Fig. 3. Coefficients of predictive panel regressions for total skewness ranks

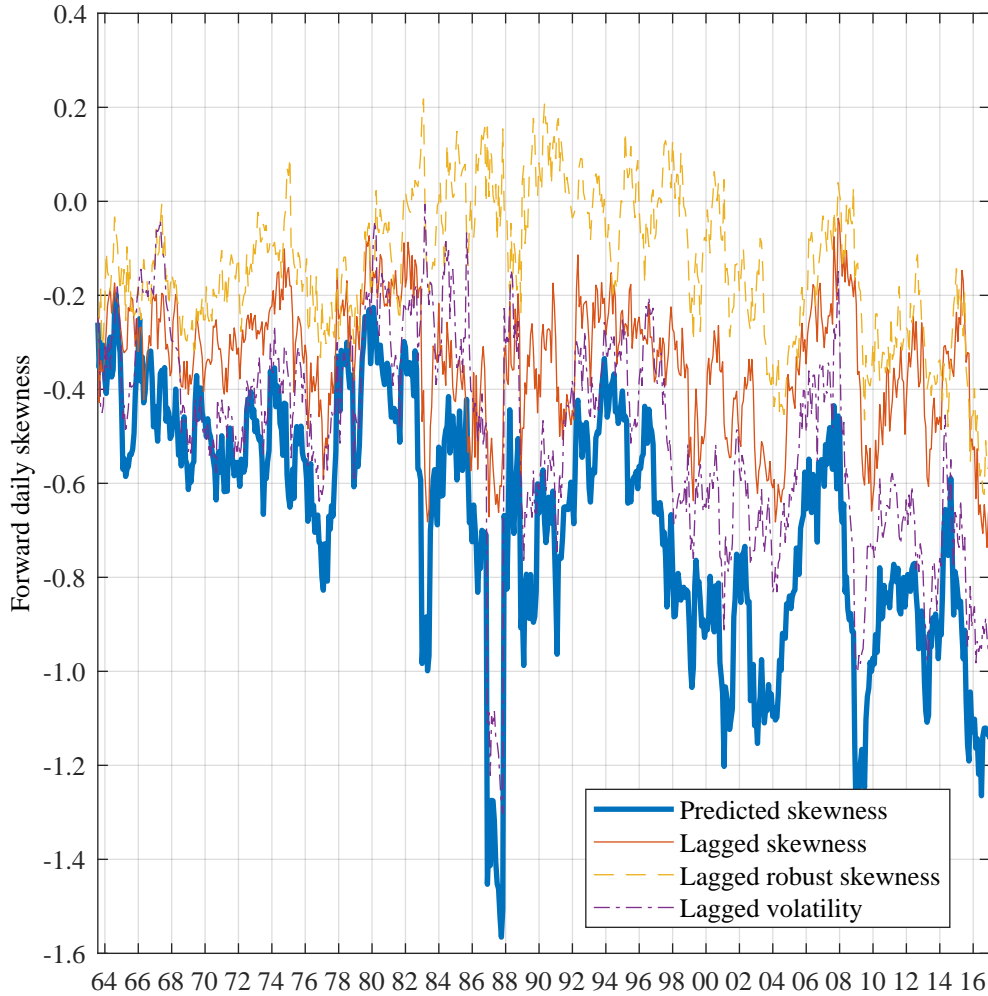
We report the panel regression coefficients  $\hat{\theta}$  and  $\hat{\phi}$  in Eq. (4) of the main text from July 1963 to December 2017. Each month, we run a panel regression that predicts the next 12-month realized daily total skewness using past risk measures and stock characteristics. We use the cross-sectional rank of total skewness as the dependent variable and the cross-sectional ranks of past risk measures and characteristics as predictors. We use all past observations to estimate at each point in time the panel regression. The construction of all variables is detailed in Appendix A of the main text.

Table 3: Summary statistics for panel regression coefficients to predict total skewness ranks

	Average	5 <sup>th</sup> percentile	95 <sup>th</sup> percentile
Market beta $\beta_M$	0.015	0.005	0.024
Idiosyncratic volatility	0.089	0.040	0.117
Coskewness $Cos$	0.003	-0.012	0.041
Idiosyncratic skewness	0.121	0.110	0.131
Market capitalization	-0.232	-0.294	-0.193
Book-to-price ratio	0.033	0.017	0.062
Net payout yield	-0.038	-0.111	-0.015
Profitability	-0.011	-0.023	0.031
Investment	-0.034	-0.043	-0.026
Momentum	-0.096	-0.114	-0.072
Intermediate horizon return	0.006	-0.007	0.014
Lagged monthly return	-0.034	-0.046	-0.025
Price impact	-0.021	-0.059	0.008
Turnover	-0.014	-0.042	0.018
Maximum return	-0.005	-0.024	0.033

We report summary statistics of panel regression coefficients  $\hat{\theta}$  and  $\hat{\phi}$  from July 1963 to December 2017. We compute the time-series average and 5<sup>th</sup> and 95<sup>th</sup> percentiles. Each month, we run a panel regression that predicts the cross-sectional rank of the daily total skewness computed over the next year using past risk measures and stock characteristics. We use the cross-sectional rank of total skewness as the dependent variable and the cross-sectional ranks of past risk measures and characteristics as predictors. We use all past observations to estimate at each point in time the panel regression. The construction of all variables is detailed in Appendix A of the main text.





**Fig. 4. Equal-weighted average of stock-specific realized total skewness**

We report equal-weighted averages of stock-specific realized total skewness. Each month, we rank stocks based on a predictor of future total skewness. As predictors, we use daily return total skewness computed over the last year, the daily quantile-based skewness computed over the last year, the daily volatility computed over the last year, and the panel regression forecasted total skewness cross-sectional ranks. We then compute each stock's daily return total skewness over the next year. For each predictor and each month, we report the equal-weighted average total skewness of the bottom 30% stocks minus the equal-weighted average total skewness of the top 30% stocks. The sample period is July 1963 to December 2017.

## 5. Robustness checks for skewness sorted portfolios

In this section, we report additional robustness checks on the risk-adjusted performance of skewness sorted portfolios. Tables 4-7 report factor spanning tests for portfolios sorted by a quantile-based measure of skewness. Table 4 uses equal-weighted portfolios sorted by predicted quantile-based idiosyncratic skewness whereas Table 5 reports on value-weighted portfolios. Table 6 uses equal-weighted portfolios sorted by predicted quantile-based total skewness whereas Table 7 reports on value-weighted portfolios.

Table 4: Factor analysis of equal-weighted portfolios sorted by predicted quantile-based idiosyncratic skewness

Portfolio	$\alpha(\%)$	$\beta_{MKT}$	$\beta_{PSS}$	$\beta_{HML}$	$\beta_{MOM}$	$\beta_{RMW}$	$\beta_{CMA}$	Adj. $R^2$
<i>Panel A: MKT, PSS</i>								
Low	0.24 (3.02)	0.76 (25.87)	-0.06 (-1.09)					0.82
Medium	0.24 (2.50)	1.05 (38.91)	0.17 (2.42)					0.87
High	-0.13 (-0.91)	1.33 (26.80)	0.61 (9.22)					0.77
Low-High	0.37 (2.61)	-0.57 (-9.21)	-0.67 (-9.90)					0.52
<i>Panel B: MKT, PSS, HML, MOM</i>								
Low	0.12 (2.02)	0.80 (38.18)	-0.03 (-0.98)	0.28 (5.64)	-0.00 (-0.10)			0.87
Medium	0.24 (3.05)	1.04 (46.33)	0.25 (5.71)	0.24 (5.19)	-0.18 (-6.94)			0.91
High	0.20 (1.24)	1.22 (28.31)	0.83 (14.77)	0.04 (0.47)	-0.59 (-7.46)			0.85
Low-High	-0.09 (-0.53)	-0.41 (-8.71)	-0.87 (-13.74)	0.24 (2.21)	0.59 (6.07)			0.67
<i>Panel C: MKT, PSS, HML, RMW, CMA</i>								
Low	-0.03 (-0.54)	0.84 (43.27)	0.03 (1.07)	0.19 (5.54)		0.25 (6.56)	0.23 (4.59)	0.89
Medium	0.04 (0.50)	1.11 (54.15)	0.22 (4.85)	0.25 (5.20)		0.10 (1.90)	0.11 (1.74)	0.89
High	0.02 (0.11)	1.31 (28.75)	0.49 (5.45)	0.22 (2.30)		-0.57 (-4.23)	-0.09 (-0.43)	0.79
Low-High	-0.05 (-0.24)	-0.47 (-9.43)	-0.46 (-5.09)	-0.04 (-0.34)		0.82 (5.31)	0.32 (1.39)	0.59

We run time-series regressions of portfolio excess returns on different factor models. Each month, we run a panel regression that predicts the next 12-month realized daily quantile-based idiosyncratic skewness using past risk measures and stock characteristics (see Appendix A of the main text). We use the cross-sectional rank of quantile-based idiosyncratic skewness as the dependent variable and the cross-sectional ranks of past risk measures and characteristics as predictors. We form equal-weighted portfolios: one with the bottom 30% stocks with the lowest-predicted quantile-based idiosyncratic skewness ranks (Low), one with the middle 40% stocks (Medium), one with the top 30% stocks with the highest-predicted quantile-based idiosyncratic skewness ranks (High), and a low-minus-high portfolio. We use the CAPM augmented with the predicted systematic skewness factor (*PSS*) in Panel A, the modified four-factor model with *MKT*, *PSS*, value (*HML*), and momentum (*MOM*) factors in Panel B, and the modified five-factor model with *MKT*, *HML*, *PSS*, profitability (*RMW*), and investment (*CMA*) factors in Panel C. For each regression, we report the monthly  $\alpha$  in %, the factor exposures, and adjusted  $R^2$ . We report in parentheses the  $t$ -ratios using a Newey-West estimator with  $T^{0.25} \approx 6$  lags. The data are monthly from July 1963 to December 2017.

Table 5: **Factor analysis of value-weighted portfolios sorted by predicted quantile-based idiosyncratic skewness**

Portfolio	$\alpha(\%)$	$\beta_{MKT}$	$\beta_{PSS}$	$\beta_{HML}$	$\beta_{MOM}$	$\beta_{RMW}$	$\beta_{CMA}$	Adj. $R^2$
<i>Panel A: MKT, PSS</i>								
Low	0.16 (3.82)	0.83 (42.28)	-0.20 (-8.60)					0.93
Medium	-0.05 (-0.95)	1.16 (53.84)	0.05 (2.24)					0.94
High	-0.36 (-3.27)	1.37 (28.88)	0.47 (9.27)					0.84
Low-High	0.53 (4.16)	-0.53 (-8.70)	-0.66 (-10.66)					0.56
<i>Panel B: MKT, PSS, HML, MOM</i>								
Low	0.06 (1.48)	0.87 (70.98)	-0.23 (-11.66)	0.09 (2.64)	0.11 (5.22)			0.94
Medium	-0.04 (-0.71)	1.15 (60.08)	0.10 (4.98)	0.10 (3.48)	-0.10 (-3.85)			0.95
High	-0.18 (-1.58)	1.30 (39.44)	0.63 (11.85)	0.13 (2.14)	-0.40 (-7.82)			0.89
Low-High	0.24 (1.73)	-0.43 (-10.41)	-0.86 (-12.70)	-0.04 (-0.48)	0.51 (7.46)			0.70
<i>Panel C: MKT, PSS, HML, RMW, CMA</i>								
Low	0.00 (0.02)	0.88 (77.86)	-0.13 (-7.31)	-0.02 (-0.82)		0.24 (6.12)	0.20 (3.95)	0.95
Medium	-0.05 (-0.90)	1.16 (66.22)	0.04 (1.88)	0.16 (4.78)		-0.10 (-2.23)	-0.09 (-1.60)	0.94
High	-0.22 (-1.59)	1.34 (35.04)	0.36 (6.39)	0.35 (4.19)		-0.50 (-5.63)	-0.27 (-2.12)	0.86
Low-High	0.22 (1.28)	-0.46 (-9.96)	-0.50 (-7.07)	-0.37 (-3.66)		0.74 (6.32)	0.47 (2.79)	0.64

We run time-series regressions of portfolio excess returns on different factor models. Each month, we run a panel regression that predicts the next 12-month realized daily quantile-based idiosyncratic skewness using past risk measures and stock characteristics (see Appendix A of the main text). We use the cross-sectional rank of quantile-based idiosyncratic skewness as the dependent variable and the cross-sectional ranks of past risk measures and characteristics as predictors. We form value-weighted portfolios: one with the bottom 30% stocks with the lowest-predicted quantile-based idiosyncratic skewness ranks (Low), one with the middle 40% stocks (Medium), one with the top 30% stocks with the highest-predicted quantile-based idiosyncratic skewness ranks (High), and a low-minus-high portfolio. We use the CAPM augmented with the predicted systematic skewness factor (*PSS*) in Panel A, the modified four-factor model with *MKT*, *PSS*, value (*HML*), and momentum (*MOM*) factors in Panel B, and the modified five-factor model with *MKT*, *PSS*, *HML*, profitability (*RMW*), and investment (*CMA*) factors in Panel C. For each regression, we report the monthly  $\alpha$  in %, the factor exposures, and adjusted  $R^2$ . We report in parentheses the  $t$ -ratios using a Newey-West estimator with  $T^{0.25} \approx 6$  lags. The data are monthly from July 1963 to December 2017.

Table 6: **Factor analysis of equal-weighted portfolios sorted by predicted quantile-based skewness**

Portfolio	$\alpha$ (%)	$\beta_{MKT}$	$\beta_{PSS}$	$\beta_{HML}$	$\beta_{MOM}$	$\beta_{RMW}$	$\beta_{CMA}$	Adj. $R^2$
<i>Panel A: MKT, PSS</i>								
Low	0.25 (3.00)	0.74 (24.10)	-0.02 (-0.46)					0.80
Medium	0.25 (2.59)	1.05 (39.80)	0.16 (2.40)					0.87
High	-0.15 (-1.04)	1.35 (26.80)	0.59 (8.98)					0.77
Low-High	0.40 (2.79)	-0.62 (-9.69)	-0.61 (-9.19)					0.51
<i>Panel B: MKT, PSS, HML, MOM</i>								
Low	0.12 (1.96)	0.78 (33.93)	0.00 (0.13)	0.30 (6.09)	-0.01 (-0.34)			0.85
Medium	0.24 (3.10)	1.05 (48.86)	0.25 (5.62)	0.24 (5.02)	-0.17 (-6.78)			0.91
High	0.20 (1.19)	1.24 (28.65)	0.81 (14.53)	0.02 (0.29)	-0.59 (-7.32)			0.86
Low-High	-0.07 (-0.44)	-0.46 (-9.42)	-0.81 (-12.91)	0.27 (2.52)	0.58 (5.85)			0.67
<i>Panel C: MKT, PSS, HML, RMW, CMA</i>								
Low	-0.03 (-0.46)	0.82 (37.95)	0.06 (2.08)	0.20 (5.33)		0.23 (6.25)	0.24 (4.82)	0.87
Medium	0.05 (0.55)	1.11 (57.10)	0.21 (4.99)	0.25 (5.21)		0.11 (2.38)	0.10 (1.72)	0.90
High	0.02 (0.08)	1.33 (28.93)	0.46 (5.12)	0.21 (2.26)		-0.58 (-4.26)	-0.10 (-0.49)	0.79
Low-High	-0.05 (-0.21)	-0.51 (-9.83)	-0.40 (-4.49)	-0.02 (-0.16)		0.82 (5.30)	0.35 (1.52)	0.59

We run time-series regressions of portfolio excess returns on different factor models. Each month, we run a panel regression that predicts the next 12-month realized daily quantile-based skewness using past risk measures and stock characteristics (see Appendix A of the main text). We use the cross-sectional rank of quantile-based skewness as the dependent variable and the cross-sectional ranks of past risk measures and characteristics as predictors. We form equal-weighted portfolios: one with the bottom 30% stocks with the lowest-predicted quantile-based skewness ranks (Low), one with the middle 40% stocks (Medium), one with the top 30% stocks with the highest-predicted quantile-based skewness ranks (High), and a low-minus-high portfolio. We use the CAPM augmented with the predicted systematic skewness factor (*PSS*) in Panel A, the modified four-factor model with *MKT*, *PSS*, value (*HML*), and momentum (*MOM*) factors in Panel B, and the modified five-factor model with *MKT*, *PSS*, *HML*, profitability (*RMW*), and investment (*CMA*) factors in Panel C. For each regression, we report the monthly  $\alpha$  in %, the factor exposures, and adjusted  $R^2$ . We report in parentheses the  $t$ -ratios using a Newey-West estimator with  $T^{0.25} \approx 6$  lags. The data are monthly from July 1963 to December 2017.

Table 7: **Factor analysis of value-weighted portfolios sorted by predicted quantile-based skewness**

Portfolio	$\alpha$ (%)	$\beta_{MKT}$	$\beta_{PSS}$	$\beta_{HML}$	$\beta_{MOM}$	$\beta_{RMW}$	$\beta_{CMA}$	Adj. $R^2$
<i>Panel A: MKT, PSS</i>								
Low	0.18 (3.95)	0.81 (36.95)	-0.19 (-6.92)					0.91
Medium	-0.03 (-0.61)	1.14 (53.19)	-0.01 (-0.48)					0.94
High	-0.40 (-3.62)	1.39 (27.98)	0.39 (7.41)					0.84
Low-High	0.58 (4.41)	-0.59 (-8.83)	-0.57 (-8.26)					0.53
<i>Panel B: MKT, PSS, HML, MOM</i>								
Low	0.05 (1.18)	0.85 (60.80)	-0.22 (-9.72)	0.11 (2.92)	0.12 (4.56)			0.93
Medium	-0.02 (-0.33)	1.13 (59.87)	0.03 (1.54)	0.09 (3.34)	-0.09 (-3.51)			0.95
High	-0.22 (-1.85)	1.33 (35.84)	0.54 (8.79)	0.10 (1.37)	-0.39 (-6.32)			0.88
Low-High	0.27 (1.84)	-0.48 (-10.09)	-0.76 (-9.60)	0.01 (0.13)	0.51 (6.08)			0.66
<i>Panel C: MKT, PSS, HML, RMW, CMA</i>								
Low	-0.01 (-0.17)	0.86 (65.96)	-0.11 (-5.38)	0.00 (0.08)		0.27 (6.20)	0.19 (3.40)	0.93
Medium	-0.05 (-0.77)	1.14 (59.34)	-0.02 (-0.93)	0.14 (3.57)		-0.06 (-1.77)	-0.07 (-1.21)	0.94
High	-0.23 (-1.60)	1.36 (34.14)	0.28 (4.83)	0.33 (3.68)		-0.52 (-5.71)	-0.30 (-2.36)	0.86
Low-High	0.22 (1.21)	-0.50 (-10.10)	-0.39 (-5.32)	-0.32 (-2.95)		0.79 (6.32)	0.50 (2.84)	0.62

We run time-series regressions of portfolio excess returns on different factor models. Each month, we run a panel regression that predicts the next 12-month realized daily quantile-based skewness using past risk measures and stock characteristics (see Appendix A of the main text). We use the cross-sectional rank of quantile-based skewness as the dependent variable and the cross-sectional ranks of past risk measures and characteristics as predictors. We form value-weighted portfolios: one with the bottom 30% stocks with the lowest-predicted quantile-based skewness ranks (Low), one with the middle 40% stocks (Medium), one with the top 30% stocks with the highest-predicted quantile-based skewness ranks (High), and a low-minus-high portfolio. We use the CAPM augmented with the predicted systematic skewness factor (*PSS*) in Panel A, the modified four-factor model with *MKT*, *PSS*, value (*HML*), and momentum (*MOM*) factors in Panel B, and the modified five-factor model with *MKT*, *PSS*, *HML*, profitability (*RMW*), and investment (*CMA*) factors in Panel C. For each regression, we report the monthly  $\alpha$  in %, the factor exposures, and adjusted  $R^2$ . We report in parentheses the  $t$ -ratios using a Newey-West estimator with  $T^{0.25} \approx 6$  lags. The data are monthly from July 1963 to December 2017.

## 6. A predicted covariance factor

In this section, we form a Predicted Systematic Covariance factor (*PSC*) and examine its importance in asset pricing tests. We first verify that *PSC* creates a significant spread in realized covariances. Second, we run all asset pricing tests contained in Section 3 of the paper replacing the market portfolio *MKT* by *PSC*. We obtain similar results; the risk premium for *PSC* is small and almost always insignificant. To justify these empirical results, we derive a stylized Three-moment CAPM in Section 7 to show that an expected-return-to-beta slope smaller than the market risk premium is expected in such a model.

First, we run the predictive panel regression in Eq. (4) in the main text with the future daily covariance on the left hand side. As with skewness measures, we measure covariance using daily returns over a 12-month period. Table 8 reports the summary statistics of different high-minus-low covariance factors along with the value-weighted market portfolio in the last line. This Table is directly comparable to Table 1 in the main text except that we replace the measure of standardized systematic skewness measure in the sixth column by covariance.

We compare the predicted covariance factor (see *PSC* on the third line) with two other factors built with other predictors of covariance: the first using covariances computed over the last 60 monthly returns and the second using covariances computed on daily returns over the previous 12 months. In all cases, we buy each month a value-weighted portfolio with the top 30% covariance stocks and short-sell a value-weighted portfolio with the bottom 30% covariance stocks.

All realized covariance measures reported in the sixth column are positive and significant, and close to each other. Covariances are much more persistent than skewness measures are, and accordingly our predictive panel regression approach does not add as much value for covariance as for skewness measures.

Next, we reproduce Tables 2-5 of the main text in Tables 9-12 replacing the value-weighted market portfolio excess return *MKT* by *PSC*.

Across all estimations in Tables 2 to 5 in main text, the risk premium for *MKT* is significantly negative in three out 24 cases and insignificant in the other 21 cases. The significantly negative risk premium estimates for *MKT* in Table 2 in the main text are likely explained by omitted factors. Indeed, the risk premium is insignificant in most cases in Tables 3 and 4 and the risk premium is

insignificant when we use the Giglio and Xiu (2017) methodology that controls for omitted factors and measurement errors in Table 5.

Across all estimations using *PSC* in Tables 9-12, the risk premium for *PSC* is significantly negative in eight out of 24 cases and insignificant in the other 16 cases. In particular, the risk premium estimates based on the Giglio and Xiu (2017) methodology are always insignificant. Hence, it appears that using a high-minus-low  $\beta$  factor unfortunately does not lead to higher risk premium estimates.

In the next section, we provide a stylized Three-moment CAPM to explain why we should expect a positive and significant risk premium for coskewness and a lower and potentially insignificant risk premium for the market portfolio.



Table 8: Summary statistics for factors built from different covariance measures

Factor	Annualized average excess return	Annualized volatility	Sharpe ratio	$\beta_M$	Covariance ( $\times 1000$ )	Annualized CAPM $\alpha$	Annualized Four-factor $\alpha$	Annualized Five-factor $\alpha$
Monthly <i>Cov</i>	-0.13	16.67	-0.01	0.71**	1.37**	-4.66*	-1.55	-0.90
Daily <i>Cov</i>	-0.17	15.91	-0.01	0.68**	1.32**	-4.53**	-2.17	-1.13
<i>PSC</i>	-1.47	14.91	-0.10	0.66**	1.26**	-5.64**	-3.80*	-2.41
Market	6.37**	15.20	0.42					

We report summary statistics of monthly returns of different covariance factors and the value-weighted market portfolio from July 1963 to December 2017. We report the annualized average return (in %), volatility (in %), and Sharpe ratio. Next, we report the market  $\beta_M$  and the covariance with the value-weighted market portfolio. Finally, we report the regression  $\alpha$  (annualized in %) using different factor models. Each month and for each covariance measure, we compute the return of a factor that is long a value-weighted portfolio containing the stocks with the top 30% values and short a value-weighted portfolio containing the stocks with the lowest 30% values. Monthly *Cov* is the covariance between monthly stock returns and market returns over the past 60 months. Daily *Cov* is the covariance between daily stock returns and market returns over the past 12 months. Each month, we run the panel regression in Eq. (4) of the main text, to predict the cross-sectional rank of future covariance using cross-sectional ranks of predictors on the right-hand side. We form the predicted systematic covariance factors *PSC* by forming a value-weighted portfolio with the stocks with the top 30% predicted covariance ranks and shorting a value-weighted portfolio with the stocks with the bottom 30% predicted covariance ranks. The market return is the value-weighted portfolio of all stocks on NYSE, AMEX, and NASDAQ. For each factor and the market portfolio, we simulate 10,000 samples under the null hypothesis from a bivariate normal distribution with the same means and variances and zero correlation, and compute covariances to obtain the significance level. All other significance levels are obtained using a Newey-West estimator with  $T^{0.25} \approx 6$  lags. \* and \*\* denote significance at the 5% and 1% level, respectively.

Table 9: Is the *PSS* factor significant?

Model		<i>Constant</i>	<i>PSC</i>	<i>PSS</i>	<i>PSC</i> <sup>-</sup>	<i>PSC</i> <sup>+</sup>	<i>OLS R</i> <sup>2</sup>	<i>GLS R</i> <sup>2</sup>
<i>Panel A: 25 Size and Book-to-price Ratio Portfolios</i>								
<i>PSC</i>	Risk premium	1.17 (6.41)	-0.90 (-3.12)				0.17	0.14
	Price of risk	0.01 (6.41)	-4.85 (-3.19)					
<i>PSC, PSS</i>	Risk premium	1.32 (6.84)	-1.14 (-3.98)	0.57 (2.41)			0.67 (0.02)	0.31 (0.01)
	Price of risk	0.01 (6.84)	-7.21 (-4.23)	5.10 (2.69)				
<i>PSS, β<sub>PSC</sub><sup>-</sup>, β<sub>PSC</sub><sup>+</sup></i>	Risk premium	1.32 (6.77)		0.55 (2.31)	-0.76 (-1.95)	-0.34 (-0.72)	0.68 (0.95)	0.33 (0.62)
	Price of risk	0.01 (6.77)		5.16 (2.80)	-12.49 (-1.90)	-3.37 (-0.70)		
<i>Panel B: 25 Size and Momentum Portfolios</i>								
<i>PSC</i>	Risk premium	0.94 (5.30)	-0.56 (-2.00)				0.07	0.05
	Price of risk	0.01 (5.30)	-3.00 (-1.99)					
<i>PSC, PSS</i>	Risk premium	1.19 (5.27)	-0.98 (-3.10)	0.80 (3.21)			0.74 (0.00)	0.27 (0.00)
	Price of risk	0.01 (5.27)	-6.65 (-3.31)	6.31 (3.41)				
<i>PSS, β<sub>PSC</sub><sup>-</sup>, β<sub>PSC</sub><sup>+</sup></i>	Risk premium	1.18 (4.88)		0.86 (3.41)	0.43 (0.46)	-1.44 (-1.75)	0.86 (0.00)	0.32 (0.21)
	Price of risk	0.01 (4.88)		6.17 (2.95)	6.36 (0.41)	-15.43 (-1.69)		
<i>Panel C: 25 Size and Coskewness Portfolios</i>								
<i>PSC</i>	Risk premium	0.03 (0.10)	0.84 (1.56)				0.94	0.14
	Price of risk	0.00 (0.10)	4.56 (1.55)					
<i>PSC, PSS</i>	Risk premium	0.35 (0.97)	0.33 (0.58)	0.47 (2.94)			0.98 (0.13)	0.31 (0.04)
	Price of risk	0.00 (0.97)	1.25 (0.39)	2.51 (2.01)				
<i>PSS, β<sub>PSC</sub><sup>-</sup>, β<sub>PSC</sub><sup>+</sup></i>	Risk premium	0.36 (0.98)		0.47 (2.97)	-0.27 (-0.42)	0.63 (0.73)	0.98 (1.00)	0.34 (0.33)
	Price of risk	0.00 (0.98)		2.78 (2.41)	-5.53 (-0.53)	5.98 (0.68)		

We report asset pricing tests for the model with a predicted systematic covariance factor (*PSC*) and the model in which *PSC* is augmented with the predicted systematic skewness factor (*PSS*). We also report on a model where the *PSC* factor is separated into low returns (*PSC*<sup>-</sup>) and high returns (*PSC*<sup>+</sup>). We define low returns as those below the average *PSC* return minus one standard deviation. As test assets, we use 25 size and book-to-price ratio sorted U.S. equity portfolios in Panel A, 25 size and momentum sorted U.S. equity portfolios in Panel B, and 25 size and predicted coskewness sorted U.S. equity portfolios in Panel C. For each test portfolio, we run a time-series regression of its excess returns on a constant and factor returns. Then, we run a cross-sectional regression of average test portfolio excess returns on a constant, and betas obtained from the time-series regressions. For each model, we estimate the beta form and report the risk premia in % per month for each factor in the first two rows. We then estimate the model in its covariance form and report the prices of risk in the last two rows for each model. We report the model misspecification robust *t*-ratios from Kan, Robotti, and Shanken (2013) below risk premia and prices of risk. Below the *R*<sup>2</sup>s for the “*PSC, PSS*” model, we report the *p*-value for the one-sided test that the model has a significantly higher *R*<sup>2</sup> than the “*PSC*” model. Below the *R*<sup>2</sup>s for the last model, we report the *p*-value for the two-sided test for non-nested models that the model has a significantly different *R*<sup>2</sup> than the “*PSC, PSS*” model (see Kan et al., 2013). The data are monthly from July 1963 to December 2017.

Table 10: Is the *PSS* factor significant in the four-factor model?

Model	Constant	<i>PSC</i>	<i>PSS</i>	<i>SMB</i>	<i>HML</i>	<i>MOM</i>	<i>OLS R</i> <sup>2</sup>	<i>GLS R</i> <sup>2</sup>
<i>Panel A: 25 Size and Book-to-price Ratio Portfolios</i>								
<i>PSC, SMB, HML, MOM</i>	Risk premium	1.00 (3.57)	-0.44 (-0.79)	0.22 (1.73)	0.35 (2.66)	1.64 (1.34)	0.75	0.40
	Price of risk	0.01 (3.57)	-0.54 (-0.11)	3.87 (1.99)	7.71 (1.56)	10.22 (1.30)		
<i>PSC, PSS, HML, MOM</i>	Risk premium	1.06 (3.73)	-0.51 (-0.95)	0.99 (2.13)	0.35 (2.67)	1.66 (1.41)	0.75 (1.00)	0.41 (0.69)
	Price of risk	0.01 (3.73)	-1.39 (-0.29)	4.18 (1.90)	7.17 (1.49)	8.96 (1.18)		
<i>Panel B: 25 Size and Momentum Portfolios</i>								
<i>PSC, SMB, HML, MOM</i>	Risk premium	1.49 (3.55)	-1.40 (-2.24)	0.46 (2.80)	-0.65 (-1.08)	0.68 (3.71)	0.87	0.36
	Price of risk	0.01 (3.55)	-11.75 (-1.83)	6.57 (2.21)	-13.14 (-1.12)	1.51 (0.57)		
<i>PSC, PSS, HML, MOM</i>	Risk premium	1.57 (3.43)	-1.53 (-2.23)	0.76 (2.92)	-0.49 (-0.87)	0.69 (3.79)	0.88 (0.77)	0.35 (0.64)
	Price of risk	0.02 (3.43)	-12.36 (-1.81)	6.11 (2.08)	-11.66 (-1.02)	-0.20 (-0.07)		
<i>Panel C: 25 Size and Coskewness Portfolios</i>								
<i>PSC, SMB, HML, MOM</i>	Risk premium	0.28 (0.75)	0.36 (0.60)	0.51 (2.97)	0.14 (0.48)	-0.30 (-0.87)	0.98	0.40
	Price of risk	0.00 (0.75)	1.86 (0.50)	5.46 (2.48)	3.71 (0.87)	-1.12 (-0.59)		
<i>PSC, PSS, HML, MOM</i>	Risk premium	0.47 (1.15)	0.05 (0.08)	0.48 (2.98)	0.12 (0.40)	-0.42 (-1.07)	0.98 (0.96)	0.39 (0.82)
	Price of risk	0.00 (1.15)	-0.53 (-0.12)	4.28 (2.24)	1.27 (0.31)	-3.60 (-1.26)		

We report asset pricing tests for the four-factor model with market (*PSC*), size (*SMB*), value (*HML*), and momentum (*MOM*) factors and a four-factor model with *PSC*, *PSS*, *HML*, and *MOM*. As test assets, we use 25 size and book-to-price ratio sorted U.S. equity portfolios in Panel A, 25 size and momentum sorted U.S. equity portfolios in Panel B, and 25 size and predicted coskewness sorted U.S. equity portfolios in Panel C. For each test portfolio, we run a time-series regression of its excess returns on a constant and factor returns. Then, we run a cross-sectional regression of average test portfolio excess returns on a constant and betas obtained from the time-series regressions. For each model, we estimate the beta form and report the risk premia in % per month for each factor in the first two rows. We then estimate the model in its covariance form and report the prices of risk in the last two rows for each model. We report the model misspecification robust *t*-ratios from Kan et al. (2013) below risk premia and prices of risk. Below the *R*<sup>2</sup>s for the last model, we report the *p*-value for the two-sided test for non-nested models that the model has a significantly different *R*<sup>2</sup> than the four-factor model (see Kan et al., 2013). The data are monthly from July 1963 to December 2017.

Table 11: Is the *PSS* factor significant in the five-factor model?

Model	<i>Constant</i>	<i>PSC</i>	<i>PSS</i>	<i>SMB<sub>FF5</sub></i>	<i>HML</i>	<i>RMW</i>	<i>CMA</i>	<i>OLS R<sup>2</sup></i>	<i>GLS R<sup>2</sup></i>
<i>Panel A: 25 Size and Book-to-price Ratio Portfolios</i>									
<i>PSC, SMB<sub>FF5</sub>, HML, RMW, CMA</i>	Risk premium	1.12 (5.22)	-0.84 (-2.47)	0.26 (2.04)	0.34 (2.59)	0.23 (0.99)	0.18 (0.71)	0.78	0.31
	Price of risk	0.01 (5.22)	-4.60 (-1.59)	5.58 (3.15)	2.94 (0.48)	5.41 (1.09)	-2.14 (-0.17)		
<i>PSC, PSS, HML, RMW, CMA</i>	Risk premium	1.10 (5.33)	-0.67 (-2.02)	0.76 (2.83)	0.34 (2.58)	0.33 (1.76)	0.21 (0.98)	0.84 (0.15)	0.46 (0.00)
	Price of risk	0.01 (5.33)	-2.75 (-0.91)	9.23 (4.06)	2.44 (0.45)	13.52 (2.62)	4.10 (0.35)		
<i>Panel B: 25 Size and Momentum Portfolios</i>									
<i>PSC, SMB<sub>FF5</sub>, HML, RMW, CMA</i>	Risk premium	1.33 (2.44)	-0.92 (-1.06)	0.42 (2.87)	-0.75 (-1.84)	0.16 (0.38)	0.39 (0.89)	0.87	0.44
	Price of risk	0.01 (2.44)	-4.41 (-0.51)	8.72 (2.66)	-29.63 (-2.19)	9.32 (0.71)	35.81 (1.47)		
<i>PSC, PSS, HML, RMW, CMA</i>	Risk premium	1.30 (2.34)	-0.89 (-1.02)	0.65 (2.43)	-0.26 (-0.60)	-0.05 (-0.13)	0.57 (1.27)	0.88 (0.86)	0.43 (0.84)
	Price of risk	0.01 (2.34)	-2.91 (-0.33)	6.92 (2.33)	-20.36 (-1.58)	6.49 (0.48)	33.95 (1.41)		
<i>Panel C: 25 Size and Coskewness Portfolios</i>									
<i>PSC, SMB<sub>FF5</sub>, HML, RMW, CMA</i>	Risk premium	0.21 (0.55)	0.47 (0.76)	0.50 (2.71)	-0.04 (-0.09)	-0.02 (-0.08)	-0.25 (-0.89)	0.98	0.44
	Price of risk	0.00 (0.55)	1.15 (0.28)	5.33 (1.73)	4.93 (0.64)	1.91 (0.27)	-8.99 (-0.81)		
<i>PSC, PSS, HML, RMW, CMA</i>	Risk premium	0.27 (0.73)	0.42 (0.70)	0.47 (2.92)	-0.04 (-0.10)	-0.07 (-0.28)	-0.23 (-0.86)	0.98 (0.98)	0.35 (0.02)
	Price of risk	0.00 (0.73)	1.09 (0.27)	2.42 (1.14)	4.76 (0.51)	0.35 (0.04)	-8.24 (-0.62)		

We report asset pricing tests for the five-factor model with market (*PSC*), size (*SMB<sub>FF5</sub>*), value (*HML*), profitability (*RMW*), and investment (*CMA*) factors and a five-factor model with *PSC*, *PSS*, *HML*, *RMW*, and *CMA*. As test assets, we use 25 size and book-to-price ratio sorted U.S. equity portfolios in Panel A, 25 size and momentum sorted U.S. equity portfolios in Panel B, and 25 size and predicted coskewness sorted U.S. equity portfolios in Panel C. For each test portfolio, we run a time-series regression of its excess returns on a constant and factor returns. Then, we run a cross-sectional regression of average test portfolio excess returns on a constant and betas obtained from the time-series regressions. For each model, we estimate the beta form and report the risk premia in % per month for each factor in the first two rows. We then estimate the model in its covariance form and report the prices of risk in the last two rows for each model. We report the model misspecification robust *t*-ratios from Kan et al. (2013) below risk premia and prices of risk. Below the *R*<sup>2</sup>s for the last model, we report the *p*-value for the two-sided test for non-nested models that the model has a significantly different *R*<sup>2</sup> than the five-factor model (see Kan et al., 2013). The data are monthly from July 1963 to December 2017.

Table 12: **Robustness check - Asset pricing tests with omitted factors**

Test Portfolios	$p$	$Constant$	$PSC$	$PSS$
75 Size, Book-to-price ratio, Momentum, and Coskewness Portfolios	4	0.38 (2.02)	-0.08 (-0.47)	0.45 (2.83)
	5	0.37 (1.88)	-0.08 (-0.43)	0.44 (2.69)
	6*	0.46 (1.73)	-0.13 (-0.62)	0.44 (2.74)
202 Portfolios from Giglio and Xiu (2017)	4	0.21 (2.26)	-0.16 (-1.04)	0.28 (1.89)
	5	0.26 (2.52)	-0.18 (-1.15)	0.30 (2.00)
	6*	0.21 (1.69)	-0.15 (-0.94)	0.30 (1.99)

We report estimated constants and risk premia for the model with a predicted systematic covariance factor ( $PSC$ ) and a predicted systematic skewness factor ( $PSS$ ). We use the estimation methodology of Giglio and Xiu (2017) which is robust to omitted factors and measurement error. As test assets, we use 25 size and book-to-price ratio, 25 size and momentum, and 25 size and predicted coskewness sorted U.S. equity portfolios in the top rows. We use the 202 U.S. equity portfolios from Giglio and Xiu (2017) in the bottom rows (25 portfolios sorted by size and book-to-market ratio, 17 industry portfolios, 25 portfolios sorted by operating profitability and investment, 25 portfolios sorted by size and variance, 35 portfolios sorted by size and net issuance, 25 portfolios sorted by size and accruals, 25 portfolios sorted by size and momentum, and 25 portfolios sorted by size and beta). The second column reports on the number of principal components used to span the space of asset returns. The star denotes the optimal number as identified by their methodology. The last three columns report on the constant, the risk premium for  $PSC$  and the risk premium for  $PSS$ , all reported in % per month.  $t$ -ratios are below risk premia in parentheses. The data are monthly from July 1963 to December 2017.

## 7. Why is the expected return-to-market beta relation small?

In this section, we derive a Three-moment CAPM under specific assumptions. We use this model to justify the empirical results we obtain, namely an insignificant market risk premium and a significantly positive coskewness premium.

We consider an overlapping-generation (OLG) economy in which investors  $i = 1, \dots, I$  are born each time period  $t$  with wealth  $W_{i,t}$  and live for two periods. Investors trade stocks  $n = 1, \dots, N$  whose returns follow a mixed Normal-exponential distribution as in [Dahlquist, Farago, and Tedongap \(2017\)](#)

$$r_{t+1} - r_f = \mu_{t+1} + \sigma_{t+1} \circ \lambda_{t+1} (g_{t+1} - 1) + \text{diag} \left( \sigma_{t+1} \circ \sqrt{\iota - \lambda_{t+1} \circ \lambda_{t+1}} \right) z_{t+1}, \quad z_t \sim N(0, C_{t+1}), \quad (1)$$

where  $r_{t+1}$  is a  $N$ -by-one vector of stock returns,  $r_f$  is the risk free rate,  $\mu_{t+1} = E_t[r_{t+1} - r_f]$  is a  $N$ -by-one vector of expected excess returns,  $g_{t+1}$  is a common shock that follows an exponential distribution with parameter 1,  $\lambda_{t+1}$  is a  $N$ -by-one vector of stock exposures to the common shock  $g_{t+1}$ ,  $\sigma_{t+1}$  is a  $N$ -by-one vector of stock return volatilities, the  $N$ -by-one vector of random variables  $z_{t+1}$  is normally distributed with correlation matrix  $C_{t+1}$ ,  $\iota$  is a  $N$ -by-one vector of ones, and  $\circ$  denotes the element-by-element multiplication. In this return model, the return skewness is  $2\lambda_{t+1}^3$ .

In time period  $t$ , investor  $i$  chooses a portfolio with weights  $\omega_{i,t}$  in stocks and the rest invested in the risk-free asset to maximize his expected utility

$$\begin{aligned} \max_{\omega_{i,t}} \quad & E_t \left[ U_i \left( W_{i,t} \left( 1 + r_f + \omega_{i,t}^\top (r_{t+1} - r_f) \right) \right) \right] \\ \text{s.t.} \quad & \omega_{i,t}^\top \iota \leq L_{i,t}. \end{aligned} \quad (2)$$

Following [Frazzini and Pedersen \(2014\)](#), investors differ in the amount of leverage they can use in their portfolio. An investor with leverage constraint  $L_{i,t} \leq 1$  cannot take any leverage while an investor with margin constraint  $L_{i,t} > 1$  can use leverage. For instance,  $L_{i,t} = 2$  implies that investor  $i$  can borrow up to his wealth to leverage his investment.

The first order conditions for investor  $i$ 's portfolio allocation are

$$W_{i,t} \text{Cov}_t \left( U_i'(W_{i,t+1}), r_{t+1} - r_f \right) + W_{i,t} E_t \left[ U_i'(W_{i,t+1}) \right] \mu_{t+1} - \psi_{i,t} = 0, \quad (3)$$

where  $\psi_{i,t}$  is the Lagrange multiplier for the portfolio leverage constraint. We solve for the competitive market equilibrium to obtain Proposition 1.

**Proposition 1.** *In equilibrium, the vector of expected excess returns is given by*

$$\mu_{t+1} = \theta_t \psi_t (\iota - \beta_{t+1}) + \beta_{t+1} (\mu_{M,t+1} - \theta_t B_t) + \theta_t B_t \frac{\text{Cov}_t(r_{t+1}, r_{M,t+1}^2)}{\mu_{M,t+1}^3}, \quad (4)$$

where  $\beta_{t+1}$  is the vector of stock market betas, the positive constant  $\theta_t$  is a measure of global risk aversion, the nonnegative constant  $\psi_t$  is a weighted average of the Lagrange multipliers for leverage constraints,  $\mu_{M,t+1}^3$  is the third central moment of market returns, and  $B_t$  is a positive constant. The expressions for all these terms are given in the proof below.

*Proof.* See Section 7.1 □

The last term in Equation (4) is the coskewness premium. The third central moment of market returns  $\mu_M^3 = E_t [(r_{M,t+1} - \mu_{M,t})^3]$  is negative in the data. Therefore, the ratio  $\frac{\text{Cov}_t(r_{t+1}, r_{M,t+1}^2)}{\mu_{M,t+1}^3}$  is similar to a skewness beta; a higher value implies higher systematic skewness risk. When coskewness is priced, we therefore have  $B_t > 0$ .

In this model, there are two reasons why the slope of the relation between average excess returns and betas is lower than the expected excess market return. Consider first the case in which there are no leverage constraints,  $\psi_t = 0$ . In this case, the first term in the equilibrium relation,  $\theta_t \psi_t (\iota - \beta_{t+1})$ , drops out, but the second and third terms remain unchanged. Therefore, the slope of the relation between  $\mu_{i,t+1}$  and  $\beta_{i,t+1}$  across stocks is decreased by  $\theta_t B_t$ . If leverage constraints also bind in equilibrium, then this slope will be even lower because of the term  $\theta_t \psi_t \beta_{t+1}$ . Therefore, our stylized Three-Moment CAPM provides two reasons why the expected return to market beta relation estimated in the data is insignificant.

### 7.1. Proof of proportion 1

Define  $\delta_{t+1} = \sigma_{t+1} \circ \lambda_{t+1}$  for simplicity. We condition on  $g_{t+1}$  and use Stein's Lemma to simplify the covariance in Eq. (3) to obtain

$$W_{i,t} Cov_t (U'_i(W_{i,t+1}), \delta_{t+1} g_{t+1}) + W_{i,t}^2 E_t [U''_i(W_{i,t+1})] \Sigma_{t+1} \omega_{i,t} + W_{i,t} E_t [U'_i(W_{i,t+1})] \mu_{t+1} - \psi_{i,t} \iota = 0, \quad (5)$$

where  $\Sigma_{t+1} = \text{diag}(\sigma_{t+1} \circ \sqrt{\iota - \lambda_{t+1} \circ \lambda_{t+1}}) C_{t+1} \text{diag}(\sigma_{t+1} \circ \sqrt{\iota - \lambda_{t+1} \circ \lambda_{t+1}})$  is the covariance matrix of  $r_{t+1}$ .

Divide both sides by  $-W_{i,t} E_t [U''_i(W_{i,t+1})]$ , sum over all  $I$  investors, and isolate the vector of risk premia to obtain

$$\begin{aligned} \mu_{t+1} &= \theta_t^{-1} W_{M,t} Cov_t (r_{t+1}, r_{M,t+1}) - \theta_t^{-1} Cov_t \left( \sum_{i=1}^I \frac{U'_i(W_{i,t+1})}{-E_t [U''_i(W_{i,t+1})]}, \delta_{t+1} g_{t+1} \right) \\ &\quad + \theta_t^{-1} \sum_{i=1}^I \frac{\psi_{i,t}}{-E_t [U''_i(W_{i,t+1})]} \iota, \end{aligned} \quad (6)$$

where  $\theta_t^{-1} = \left( \sum_{i=1}^I \theta_{i,t}^{-1} \right)^{-1}$  and  $\theta_{i,t} = -\frac{E_t [U''_i(W_{i,t+1})]}{E_t [U'_i(W_{i,t+1})]}$  is a measure of global risk aversion (see Huang and Litzenberger, 1988, Ch. 4). Eq. (6) is also valid for the value-weighted market portfolio

$$\begin{aligned} \mu_{M,t+1} &= \theta_t^{-1} W_{M,t} Var_t (r_{M,t+1}) - \theta_t^{-1} Cov_t \left( \sum_{i=1}^I \frac{U'_i(W_{i,t+1})}{-E_t [U''_i(W_{i,t+1})]}, \delta_{M,t+1} g_{t+1} \right) \\ &\quad + \theta_t^{-1} \sum_{i=1}^I \frac{\psi_{i,t}}{-E_t [U''_i(W_{i,t+1})]}, \end{aligned} \quad (7)$$

where  $\delta_{M,t+1} = \omega_{M,t}^\top \delta_{t+1}$  and  $\omega_{M,t}$  is the  $N$ -by-one vector of market portfolio weights.

Substituting Eq. (7) into (6) yields

$$\mu_{t+1} = \theta_t^{-1} \psi_t (\iota - \beta_{t+1}) + \beta_{t+1} (\mu_{M,t+1} - \theta^{-1} B_t) + \theta_t^{-1} B_t \frac{\delta_{t+1}}{\delta_{M,t+1}}, \quad (8)$$

where  $\psi_t = \sum_{i=1}^I \frac{\psi_{i,t}}{-E_t [U''_i(W_{i,t+1})]}$  is a weighted average of the Lagrange multipliers for leverage



constraints,  $\beta_{t+1} = \frac{Cov_t(r_{t+1}, r_{M,t+1})}{Var_t(r_{M,t+1})}$  is the  $N$ -by-one vector of stock market betas, and

$$B_t = -Cov_t \left( \sum_{i=1}^I \frac{U'_i(W_{i,t+1})}{-E_t[U''_i(W_{i,t+1})]}, \delta_{M,t+1} g_{t+1} \right) \quad (9)$$

is a constant.

Proposition 1 is obtained by using the following moments of the Normal-exponential distribution

$$\begin{aligned} Cov_t(r_{t+1}, r_{M,t+1}^2) &= 2\delta_{M,t+1}^2 \delta_{t+1}, \\ E_t[(r_{M,t+1} - \mu_{M,t})^3] &= 2\delta_{M,t+1}^3. \end{aligned} \quad (10)$$

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